Name: Stephanie Morgan

Unit Plan: Week 7
Subject/Grade Level: NC Math I/9th Grade

Unit Title: Understanding Linear Functions

Standards:
North Carolina Math I Standard Course of Study
Interpreting Functions:
Interpret functions that arise in applications in terms of context

NC.M1.F-IF.6 – Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically

Analyze functions using different representations
NC.M1.F-IF.7 – Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range, rate of change, intercepts, intervals where the function is increasing, decreasing, positive, or negative, maximums & minimums, and end behaviors.

Linear, Quadratic, and Exponential Models:
Construct and compare linear and exponential models and solve problems
NC.M1.F-LE.1 – Identify situations that can be modeled with linear and exponential functions, and justify the most appropriate model for a situation based on the rate of change over equal intervals

Interpreting Categorical and Quantitative Data:
Interpret linear models
NC.M1.S-ID.7 – Interpret in context the rate of change and the intercept of a linear model. Use the linear model to interpolate and extrapolate predicted values. Assess the validity of a predicted value.

Objectives
The purpose of this lesson is to use monetary conversions as a means of teaching rates of change to students. The weights of different currencies will be based on European currencies pre-Euro, though this will not be disclosed to students until after the lesson. Students will wrestle with making conversions, and groups may find that they may not have enough money to make all of their purchases. These processes will challenge students to think critically and, in the case that a group cannot purchase all items, empathize...
with prioritization of items or how best to acquire as many items as possible. The conclusion of the lesson will include more context on what the students have just done, including the identification of the different currency types, will involve a debrief of the thought processes used and conclusions reached, and that will then connect to the idea of a monetary union and one of the motivators for the EU.

### Key Concepts

- Rate of change
- Linear functions (slope-intercept, point-slope)
- Linear applications in context
- Monetary union

### Essential Questions

- What is a rate of change?
- How is rate of change utilized in a given context?
- Explain how a rate of change connects two quantities.
- Consider the mathematical arguments for a monetary union.

### Learning Acquisition and Assessment

**Students will know... (content/concepts)**

- Rate of change
- Conversions between quantities
- Linear functions
- Slope-Intercept form
- Point-Slope form
- Currency Union

**Students will be able to... (skills, performance tasks)**

- Associate measurements of two different units to establish a rate of change between them
- Utilize rate of change to make conversions between currencies
- Represent the relationship between two quantities with tables of values, graphs, and equations
- Students will understand the x- and y-intercepts, in context, of the relationship between two quantities
- Recognize how and when to use different forms of linear functions

### Formative Assessments

- Discussion activities
- Small group activities
- Individual work

### Summative Assessments

- Homework
- Quiz

### Learning Activities (1 week – 5 days): Lesson introduction, body, and closing

#### Day 1: *Instruction*

This lesson opens up a unit on linear relationships and rates of change. Students will call upon prior knowledge of basic linear equations from 7th- and 8th-grade math courses and will build upon that here. Students will be divided into 5 groups of 6 (accounts for 30 students). The object is to allow students to engage in the Six Thinking Hats strategy of Lateral Thinking. Should a class be smaller than 30 students, then groups can be smaller and a student can wear more than one hat (though no more than 2) during the exercise. If this is the case, then when explaining the strategy, the teacher will need to carefully explain the need for altering one perspective when changing hats and potentially provide examples as a demonstration (such as changing from ‘logic’ to ‘positive’).
Once grouped, students will be provided with their currency manipulatives and an identification of their currency (Duets, Fins, Nets, Drachs, or Tonis), their thinking hats, and a handout (digital or hardcopy), that includes the following information:

**Purchasing Power**

You and your group represent the government of a nation, and it is your responsibility to acquire the goods needed through trade, namely gas, fruits and vegetables, paper, cloth, and livestock. Your trading partners are other nations with their own currencies, and you are provided with the following prices for the items you need:

<table>
<thead>
<tr>
<th></th>
<th>Fruits &amp; Vegetables</th>
<th>Paper</th>
<th>Cloth</th>
<th>Livestock</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gas</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25 Fins</td>
<td>80 Drachs</td>
<td>24 Drachs</td>
<td>15 Tonis</td>
<td>16.67 Duets</td>
</tr>
<tr>
<td><strong>37.50 Tonis</strong></td>
<td>13.33 Fins</td>
<td>2 Duets</td>
<td>8 Nets</td>
<td>26.67 Nets</td>
</tr>
</tbody>
</table>

You can only make purchases with your currency, so you must determine the how much each item will cost in your country’s currency. Your task is to determine the cost of all 5 items and, when completed, approach the teacher to make your purchases. To do so, first engage in a Six Thinking Hats discussion. Each of your group members will select a hat and, as you engage in discussion and suggest ideas, you need to follow the guidelines associated with the hat color. Those are:

- White hat: neutral and objective thinker concerned with facts and figures
- Red hat: emotional, intuitive thinker who acts on hunches and impressions
- Black hat: responsible for pointing out negative aspects (errors in logic and possible consequences)
- Yellow hat: responsible for maintaining optimism and positivity, focusing on benefits and constructive ideas
- Green hat: devising creative, innovative, and new approaches to the problem
- Blue hat: organizes thoughts and the thoughts of others

Record the wearers of the each hat in the chart below. If you have any questions about the role of the wearer of a certain hat, please ask for assistance from the teacher.
Once your group has agreed on a strategy, put it into practice. As you do so, maintain your roles based on your hats. If your conversation stalls, you may be permitted to swap hats *once*, and *everyone* in the group must assume a different hat and perspective. If this swapping of hats occurs, record the new wearers in the 2nd half of the chart below.

<table>
<thead>
<tr>
<th>Names (start of discussion)</th>
<th>Names (if swapping occurs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>White</td>
</tr>
<tr>
<td>Red</td>
<td>Red</td>
</tr>
<tr>
<td>Black</td>
<td>Black</td>
</tr>
<tr>
<td>Yellow</td>
<td>Yellow</td>
</tr>
<tr>
<td>Green</td>
<td>Green</td>
</tr>
<tr>
<td>Blue</td>
<td>Blue</td>
</tr>
</tbody>
</table>

Your group needs to show all calculations done to arrive at your conclusions for what your country will pay for your purchases. Fill in the list below when you are ready to complete your purchases and bring your list to the teacher along with the correct amount of currency (your manipulatives) along with the work you’ve done to arrive at these conclusions.

<table>
<thead>
<tr>
<th>Group Members:</th>
<th>Currency:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gas</th>
<th>Fruits &amp; Vegetables</th>
<th>Paper</th>
<th>Cloth</th>
<th>Livestock</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
Once this has been given to the groups and they have had time to review what they’re doing, the teacher should spend some time going over the Six Thinking Hats strategy. Direct student attention to the explanation of each hat wearer’s role and provide a quick example of any that the students are unsure of. Follow up with any additional questions students may have regarding the strategy during the activity. Due to the need to go through the strategy as well as potentially model the process and address any other questions, this portion of the lesson may take up to 25 minutes.

Students will probably need 35-40 minutes to complete the activity. The teacher should circulate through the room to address any questions or concerns that may arise during the activity. Students may show work written out on notebook paper, may utilize tables of values and graphs (by hand or with technology), and may use Desmos.com to help in the analysis. Groups may wish to ‘make change’ of the manipulatives, converting a whole to the fractional (cents) equivalent, which is why the tape measure and scissors are provided, as most of the manipulatives can be broken into small pieces. However students may also wish to ‘overpay’ and receive change (theoretically), and they will state such when they come to the teacher to make their final purchases. Two currencies – the Drachs and the Nets – will actually not have enough currency to make all five purchases, while the Duets will have just enough and two currencies – the Tonis and the Fins – will have excess. This is to demonstrate the power of currency and how having what appears to be an excess initially, such as 500 Drachs, may seem like a lot of money, but in relation to other currencies, it actually isn’t sufficient to acquire what is needed. These groups will still be able to complete the price conversions, but may then face needing to prioritize some items over others.

Once the activity is complete, the groups will use the self-stick wall easel pad pages to summarize their Six Thinking Hat strategy process and any big ideas, roadblocks, and conclusions they reached. Any graphs or tables of values should be included on the easel paper (if done via Desmos, groups can ask the teacher to project their graph on the board). If the groups with Drachs and Nets don’t initially bring up not having enough money, the teacher should pose a question to the class drawing this out into the conversation. As a class, this will culminate in a debriefing of the activity that will allow students to debrief the activity. Guiding questions for the teacher to jumpstart the conversation could include:

- **Nations do this every day. Describe the process you went through and what it felt like to be in this situation.**
- **Describe your frustrations with the process. Is there a way to streamline what you had to do?**
At this point, students should have brought up the idea that a common currency would have made this process much easier. When this point is reached, the teacher can show students the following chart, which lists 5 countries and their pre-Euro currencies and map:

<table>
<thead>
<tr>
<th></th>
<th>Deutschemark (Germany)</th>
<th>Markka (Finland)</th>
<th>Guilder (Netherlands)</th>
<th>Drachma (Greece)</th>
<th>Kroon (Estonia)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deutschemark</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Germany)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Markka</td>
<td>FIM:DEM 3.04:1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Finland)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Guilder</td>
<td></td>
<td>FIM:NLG 2.6981:1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Netherlands)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Drachma</td>
<td>NLG:DEM 1.12676:1</td>
<td>NLG:FIM 0.3706:1</td>
<td></td>
<td>GRD:NLG 154.6256:1</td>
<td></td>
</tr>
<tr>
<td>(Greece)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kroon</td>
<td></td>
<td></td>
<td></td>
<td>GRD:EEK 21.77700:1</td>
<td></td>
</tr>
<tr>
<td>(Estonia)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

With the support of the Erasmus+ Programme of the European Union
The actual exchange rates between these currencies were far more complicated and varied than those in the activity. The teacher can allow students to review these conversions and then ask: *Think of the frustrations you discussed earlier. How might those frustrations have changed if you were working with these factors instead?* The discussion that answers these questions can then lead into a discussion of a *monetary union*, or an agreement between two or more states to create a single currency area. Many students may have already picked upon the fact that these countries are EU nations, further supported by the map provided. The European Union should be explained as a political and economic union, which makes it a monetary union that adopted the Euro as its common currency on January 1, 1999.

Inevitably, the teacher should anticipate a student or students voicing a concern about fairness, as the purchasing power of one currency was stronger than another (and if they do not, the teacher should bring this to student attention). To further illustrate that point, the teacher can display the following table:

<table>
<thead>
<tr>
<th>→ Euro</th>
<th>Deutschmark (Germany)</th>
<th>Markka (Finland)</th>
<th>Guilder (Netherlands)</th>
<th>Drachma (Greece)</th>
<th>Kroon (Estonia)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Currency</td>
<td>Conversion Rate</td>
<td>DEM:FIM</td>
<td>DEM:NLG</td>
<td>DEM:GRD</td>
<td>DEM:EEK</td>
</tr>
<tr>
<td>-------------</td>
<td>-----------------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>Deutschemark (Germany)</td>
<td>1.95583 DEM to 1 Euro</td>
<td>0.32895:1</td>
<td>0.8875:1</td>
<td>0.00574:1</td>
<td>0.12500:1</td>
</tr>
<tr>
<td>Markka (Finland)</td>
<td>5.94573 FIM to 1 Euro</td>
<td>3.04:1</td>
<td>FIM:NLG 2.6981:1</td>
<td>FIM:GRD 0.01745:1</td>
<td>FIM:EEK 0.38001:1</td>
</tr>
<tr>
<td>Guilder (Netherlands)</td>
<td>2.20371 NLG to 1 Euro</td>
<td>NLG:DEM 1.12676:1</td>
<td>NLG:FIM 0.3706:1</td>
<td>NLG:GRD 0.006467:1</td>
<td>NLG:EEK 0.13072:1</td>
</tr>
<tr>
<td>Drachma (Greece)</td>
<td>340.75 GRD to 1 Euro</td>
<td>GRD:DEM 174.2227:1</td>
<td>GRD:FIM 57.31:1</td>
<td>GRD:NLG 154.6256:1</td>
<td>GRD:EEK 21.77700:1</td>
</tr>
<tr>
<td>Kroon (Estonia)</td>
<td>15.6464 EEK to 1 Euro</td>
<td>EEK:DEM 7.9999:1</td>
<td>EEK:FIM 2.6315:1</td>
<td>EEK:NLG 7.6497:1</td>
<td>EEK:GRD 0.04592:1</td>
</tr>
</tbody>
</table>

Clearly, the former German Deutschemark is much closer to the value of the Euro than the former Greek Drachma. This could prompt discussion initiated by the students, but if not, the teacher should ask: *Some have asserted that this is unfair, and yet these nations all agreed to adopt the Euro. Think back to your own experience with the activity and see if you can come up with any reasons why countries may still opt for a common currency.* This final part of the discussion will close out the debrief portion. All totaled, this should take about 20 minutes.

*Closing*
In the 5-10 minutes remaining, students will be provided with the document below:

This will be their assignment for the day. Even though we didn’t get the chance to review the math that they did with their groups (that will come tomorrow), they will apply the same skills to fill in the blanks here. These will be checked tomorrow and will serve as an introduction to the mathematical discussion.

Main EU-related concepts/activities: Rate of change via monetary conversions and introducing the currency situation pre-monetary union in the EU

**Day 2**

**Introduction**

To review the previous day, students will first review their work on the previous day’s assignment. Students will form groups of 2-3 and to review their work. They will be given ~5 minutes to compare answers before being provided with the following to check their work:
This will be useful because it will allow students to consider how they were doing conversions (\( \frac{\text{DKK}}{\text{USD}} \), for example) to help explain mistakes. For instance, 200 DKK * \( \frac{141745 \text{ USD}}{1 \text{ DKK}} \) does yield 28.35 USD (and does allow units – DKK – to cancel), but 200 DKK * \( \frac{1 \text{ DKK}}{141745 \text{ USD}} \) gives 1,410.98 USD, which is not what the receipt shows and, upon closer inspection, does not allow any units to cancel. All of this can be reviewed, any incorrect answers discussed among groups, and then the transition can be made to whole class discussion, which will tie into what students discovered in the previous day’s lesson. All totaled, this introduction should take about 15 minutes.

**Instruction**

The work from the introduction coupled with student graphs and tables of value from the previous day’s activity will be utilized as examples to link student thinking to the idea of *rate of change* and the construction of conversion factors. Rates of change will be connected back to Pythagorean Theorem, which student already know from earlier math courses, and right triangles. Because rates of change involve the change in y (\( \Delta y \)) being divided by the change in x (\( \Delta x \)), we often make this less formal by stating it as ‘rise over run’. In looking at the graph (below – could be drawn on a group’s self-stick page or displayed via Desmos), the line represented by the relationship between the x-axis (D1, or Duets) and the y-axis (F, or Fins), the teacher will need to be very intentional about asking students to identify units when setting up the rate of change; here, this is \( \frac{\text{rise}}{\text{run}} = \frac{+2 \text{ Fins}}{+1 \text{ Duet}} \) (up and right are both positive directions), so the rate of change is 2 Fins/Duet. When counting this, teachers need to remind students to check the scales of the axes involved, as the x- and y-axes are scaled differently here, and this is something that happens frequently. Furthermore, students need to relay their understanding of this rate of change as a conversion factor by translating it into a sentence. The teacher should ask students to do so and look for answers like:

- *For every 2 Fins, you can add 1 Duet*
- *As you increase by 1 Duet, you’re also increasing by 2 Fins.*

There are several ways that students can phrase this, and the teacher should let students discuss their suggestions with one another to see if they are saying the same thing, just altering the phrasing. If they are not saying the same thing, the teacher should let students discuss and arrive at a conclusion for how to appropriately reword the suggestion.
The example given here for Duets and Fins relates the former as the x-value and the latter as the y-value. The teacher should be prepared to discuss examples in which the two units are reversed and let students look at the phrasing of the rates of change to conclude that, even when represented differently, they are saying the same thing. However, the students should also be encouraged to relate this back to their work in the simulation, as the reversal of the two axes will not change the relationship between the two units, but will change how they are utilized in calculations. For example, in completing their exercises, the Fin group could have done the following:

**Given:** Gas = 25 Fins, Fruits & Vegetables = 13.33 Fins

**Need:** Paper, Cloth, & Livestock

To find Cloth using what was given:

\[
\frac{25 \text{ Fins}}{37.50 \text{ Tonis}} \times 15 \text{ Tonis} = \frac{375 \text{ Fins} \times \text{Tonis}}{37.50 \text{ Tonis}} = 10 \text{ Fins}
\]

- Tells us that 15 Tonis = 10 Fins = 8 Nets

To find Paper using what was given:

\[
\frac{13.33 \text{ Fins}}{80 \text{ Drachs}} \times 24 \text{ Drachs} = \frac{320 \text{ Fins} \times \text{Drachs}}{80 \text{ Drachs}} = 4 \text{ Fins}
\]

- Tells us that 24 Drachs = 4 Fins = 2 Duets

To find Livestock using what was given:

\[
\frac{4 \text{ Fins}}{2 \text{ Duets}} \times 16.67 \text{ Duets} = \frac{66.67 \text{ Fins} \times \text{Duets}}{2 \text{ Duets}} = 33.33 \text{ Fins}
\]

Here, the order in which students have organize the rates of change is important because only with the Fins unit placed on top (the ‘rise’ or Δy) can the other currency cancel out in the calculation (as it appear on both the top and bottom of the answer, which allows it to divide out).

Other groups may have chosen to find their answers using the graph, such as the Duets group. They may have done the following:
Given: Paper = 2 Duets, Livestock = 16.67 Duets
Need: Gas, Fruits & Vegetables, Cloth

To find Fruits & Vegetables using what was given: Graph $y = \frac{2}{24}x$, as shown below, to represent the relationship between Duets and Drachs:

![Graph](image)

To arrive at an answer, we must use what we know about Drachs with Fruits & Vegetables. This means Drachs are the x-value, as it takes 24 Drachs to make 2 Duets. Therefore, we graph $x = 80$ and find the intersection. The y-value of the intersection will be the answer we need.

![Graph with intersection](image)

Therefore, Fruits & Vegetables = 6.67 Duets

Similar processes would locate the remaining answers for the Duet group. What the teacher needs to ensure he or she does is make sure students recognize that, regardless of the method they use, the identification of the x- and y-axes and the placement of the units in the conversion process...
are crucially important to arriving at the correct answer. Again, with the graphing method, the teacher should still ask students to paraphrase their rates of change. For instance, in the above example, students could reply that:

- For 24 Drachs, the equivalent value is 2 Duets.
- Every 24 Drachs can be converted to 2 Duets.

Students need to see the different methods other groups utilized and teachers should ensure that they understand each process. Groups should be able to lead the discussions in these cases, particularly allowing different hat wearers to step up to explain how their viewpoint led to a particular method of solving and how they were able to make sense and keep track of the calculations as they went through. Groups can also discuss how their thinking has changed since doing the independent homework activity or, if their group work supported their independent work, how it has been strengthened. The teacher should also draw attention to the x- and y-intercepts of these graphs (the ordered pair (0,0) for both) and what that means here (ie. if you have 0 of one currency, then you have 0 of the other currency). Overall, this discussion should take between 25-30 minutes.

Now that students have had a chance to dig into the math, the discussion should transition back to the idea of a monetary union, particularly the one within the EU. In the same groups of 2-3, students should locate a Chromebook and open a copy of a Google Slide (will be found in Google Classroom). They should rename their Slide with the names of those in the group (set up to already be shared with the teacher), and then use the links provided on the first slide (see links section below) to answer the questions on the following slides. Those questions are:

Slide 2: What is the justification for the economic and monetary union (EMU) of the European Union? Its objectives?
Slide 3: Why adopt the Euro across the EU?
Slide 4: Think about your experience yesterday and today with the different conversions. How does your experience relate to the creation of the Euro and the EMU? What additional insights did you gain from the articles?

Students should spend some times reading through these articles and discussing the information they find. If they wish to explore other links connected to the sites, that should be allowed as well, but students should be reminded to ensure that they’ve answered the questions in the slides. As groups decide on their answers, one member should type them into the available text box on each slide (formatting is up to the students – should be either complete sentences or close-to-complete-sentence bullet points). Students should be given 15-20 minutes for these small-group discussions to really dig into the material.
Small-group discussions can then transition into whole-group discussion. As each question is addressed, each group should give its input, discuss any points of disagreement or different interpretations, and ensure that students understand the reasoning and objectives behind slides 2 and 3. Slide 4 is where the discussion will likely spend the most time, as this is where students can really connect their experiences from the different activities to the experience of the European Union. Each group should really take the time to report out their answers for Slide 4 and discuss their takeaways and insights from their small-scale experiences to the information gleaned from these websites. This phase of discussion should be given ~20 minutes.

Closing

In the time remaining, students should complete an Exit Ticket that consists of the following:

Your country uses Tonis as a currency. You have ancestors from the country that now uses Nets and you want to vacation there to trace your family history. Your grandmother, who supports your endeavors, gifts you with 100 Nets for your birthday.

Write an equation that would allow you to calculate how many Nets you could get back if you put in a certain number of Tonis. What is different about this equation than the ones we utilized today? If you’re struggling to come up with an equation, discuss what has you puzzled.

The teacher will collect these and analyze the answers to inform him or her about how best to approach the next day’s lesson, which will involve having an initial starting value. If students correctly identify the equation \( y = \frac{8}{15}x + 100 \), or something equivalent) and recognize that now we have to consider an initial value (the 100 Nets) and will change the x- and y-intercepts, then the teacher knows that he or she can structure the next day’s lesson differently than if students are perplexed about what the additional information represents.

Main EU-related concepts/activities: Linking rate of change with linear equations and more discussion of monetary union

Day 3

Introduction

Using the results from the previous day’s Exit Ticket, the teacher can group students to go over the question that was asked. The teacher should use students’ responses to form these groups as he or she believes will best help students discuss the prompt and their responses. Groups should be 2-4, depending on the size of the class and/or the composition of the groups. Students need to try to come to come kind of a consensus on an answer to the question, or, if a group is dealing with not being able to answer it, at least highlighting what they are having issue with. This could take ~10 minutes.

Instruction
Groups from the Introduction should report out and the class can discuss how to correctly identify the equation \( y = \frac{8}{15}x + 100 \) and why it is what it is. Special attention should be given to the identification of the intercepts and how they have changed from the other equations investigated the day before (such as \( y = \frac{2}{24}x \)). Discussion should also highlight what the y-intercept represents in the Exit Ticket question (ie. the starting amount).

At this point, the teacher should ask, “Think back to the Exit Ticket question. What significant pieces of information did we have?” Potential responses could be:

*We knew that we were looking for Tonis based on a certain number of Nets
*We had a rate of change or slope between Tonis and Nets (or \( \frac{\text{8 Nets}}{\text{15 Tonis}} \)) → if students identify the reciprocal \( \left( \frac{\text{15 Tonis}}{\text{8 Nets}} \right) \) as the rate, that can prompt discussion about units
*We have a starting value other than 0, and that serves as the y-intercept → this can lead to a discussion of the fact that the y-intercept is 0 in an equation like \( y = \frac{2}{24}x \)

This discussion should set the stage for direct instruction to come and take ~10 minutes.

The information compiled above will lead to the direct instruction on the different forms of linear equations. The first to identify is **Slope-Intercept Form**, \( y = mx + b \). Students were taught this formula in middle school math courses, but generally need a refresher at this point. Remind students that the name comes from the two things that are needed – the slope and the y-intercept. Start with examples:

\[
y = \frac{2}{24}x \quad \text{and} \quad y = \frac{8}{15}x + 100
\]

Students should be able to quickly point out the y-intercepts, particularly the 100 in the second equation, but they may need a reminder of the y-intercept (0) in the first equation. The teacher will need to explain that, generally, the 0 is not written, but that the equation could also be expressed as \( y = \frac{2}{24}x + 0 \).

Students should be asked to look about what they notice about the y-intercept and where it shows up in the equation. Students may provide several answers similar to:

*It’s the number at the end
*It’s the number without the x

Technically, as written, both of these are correct statements, but the teacher should change the order of the equations point out which statement is always correct:
y = 0 + \frac{2}{24}x \quad \text{and} \quad y = 100 + \frac{8}{15}x

In these cases, the teacher should ask: *Do both of the points above still hold true?* Obviously, the first point does not, but the second does. This is important for students to take note of – the y-intercept is *always* the number without x (or without a variable, to be more generic). This should take ~15 minutes.

So now the question becomes what does the number with x, or the *coefficient* of x, represent? The teacher will probably need to take a minute or so to be sure that students are comfortable with the terminology *coefficient* and understand that it is simply a number attached to any variable. In looking at the linear equation, the variable is generally x, and students should be able to identify that the coefficient is the rate of change, or the slope. Now, connect this back to Slope-Intercept Form, \(y = mx + b\). Ask: *Which variable represents the slope and which variable represents the y-intercept?* Based on the previous conversation, students should identify \(b\) (without the variable x) as the y-intercept, while \(m\) is the slope or rate of change (is the coefficient of x).

It should be noted that we’ve discussed rate of change or slope in earlier lessons as \(\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}\), but let’s dig into that a little further. For \(\Delta y\) to mean ‘change in y’, the teacher should ask *Can anyone think of another word that means ‘change’?* Students should be given some time to think this over independently, but then should be allowed to contribute their thoughts to the whole class. Ultimately, students should get to the idea that ‘change’ can also mean ‘different’ or, in terms of mathematics, ‘difference’. The *difference* in math is the result of a subtraction problem, so \(\Delta y = y - y\), or the subtraction of two y-values. The same can then be said for \(\Delta x\), or \(\Delta x = x - x\).

Expect student questions here. Someone will likely state that this doesn’t make sense, as \(y - y\) should be 0 and \(x - x\) should be 0. While this is potentially possible, it won’t always be the case, so we need to be able to distinguish the value. Easy fix – add subscripts so that \(\Delta y = y_2 - y_1\) and \(\Delta x = x_2 - x_1\). Now, look back at your rate of change or slope formula: \(\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = m\).

Ask students to practice. Have them answer the following questions individually:

*I still have Deutschemarks from a trip to Berlin in 2000. I also have 113 euros leftover from last summer. Knowing the conversion between Deutschemarks (DEM) and euros is 1.95583 DEM to 1 euro, write an equation in slope-intercept form that would allow me to convert my Deutschemarks.*

*I have 58 DEM. Based on the information in the previous question, how many euros do I have?*

*Answers:*
*\( y = \frac{1}{1.95583}x + 113 \)

*\( y = \frac{1}{1.95583}(58) + 113 = 142.65 \) euros

Ask students to work independently for ~5 minutes to see what they can do on their own. Then, allow students to turn and talk to someone sitting around them about their work. To allow full time for partners to discuss, expect around ~10 minutes to pass for the partner discussion. The teacher should circulate through the room to listen to the conversations taking place and consider good points or any misconceptions to address for whole-group discussion.

Once the partner discussions have concluded, the teacher should transition to whole-class discussion to talk about the answers and how they arrived at them. If no one brings up labeling x and y, the teacher should do so. For instance, the teacher should point out that if we need to convert DEM, then DEM needs to be in the denominator of the slope. It would also imply that DEM needs to be the x-value, which someone may point out (and if not, the teacher should point out) is supported by the denominator of the slope representing \( \Delta x \). This implies that \( y = \) euros, which someone may point out makes sense since it is what you’re looking to end with and is the numerator of the slope (\( \Delta y \)). The teacher should take some time to discuss the importance of labeling variables and understanding what each represents in context. This should take ~20 minutes.

Once this discussion wraps, the teacher should pose another situation to students: The exchange between Greek drachmas (GRD) and euros as 340.75 GRD to 1 euro. If you had 7.8 euros but ultimately ended with 2,800 GRD, how many GRD did you start with? Can you write an equation in slope-intercept form for this situation?

Students will need to take ~1 minute to think about what they have been given here. Then the teacher can ask for suggestions about what they think they’ve been given and how this may differ from the previous problem. Possible answers include:

*We have a rate of change/slope
*Is 2,800 GRD the y-intercept? (let students discuss this to ultimately arrive at ‘no’)
*I’m not sure what we have/We’ve got a slope but the 7.8 euros and 2,800 GRD must be something else?

This last question is especially important, and the teacher should redirect students to labeling x and y. The teacher should then give students ~1 minute to think this through independently before putting forth ideas to the class. Students are likely to say that the rate of change, based on the table from Day 1, is \( \frac{\Delta y}{\Delta x} = \frac{340.75 \text{ GRD}}{1 \text{ euro}} \), since we had 7.8 euros and ended up with 2,800 GRD, so we started with euros, which makes x = euros, so by default, y = drachmas. The
teacher should ask students what we can do with this. Again, give students time to think individually, then allow them to put forth ideas. These could include:

*If we use $y = mx + b$, then $2800 \text{ GRD} = \frac{340.75 \text{ GRD}}{1 \text{ euro}} (7.8 \text{ euros}) + b \ldots$ does that mean we have to solve for $b$? (and the answer is ‘yes’, and will imply that $b = 142.15 \text{ GRD})$

→ So what is Slope-Intercept Form? $y = \frac{340.75}{1} x + 142.15$ or $y = 340.75x + 142.15$

*Can we do this without solving for $b$? I want a quicker way.

The second question is important because some students will gravitate to the first method, but others may struggle to track from finding the $b$-value to plugging it back into a slope-intercept form. To address this question, ask students to think back to the slope equation, $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$. If we don’t like the fraction, we can multiply by the difference in the denominator, so that $y_2 - y_1 = m(x_2 - x_1)$. This is what is called Point-Slope Form. The name comes from what is needed – a point and a slope. The slope or rate of change is obvious $(340.75 \text{ GRD/euro})$, but what is the point? If $x = \text{ euro}$ and $y = \text{ drachmas}$, then the point would have been $(7.8, 2800)$. The teacher should ask: But how does that connect to the equation?

Students should be quick to point out that the $m$-value in the equation is the slope, so that takes care of one part. But what is the point? The teacher should ask Tell me about the parts of a point, or an ordered pair. Students will relate that the point is composed of $x$ and $y$, or $(x, y)$, so that should lead them to consider that the point should go in for one $x$ and one $y$, but which one?

To help students decide, the teacher should point out slope-intercept form, specifically the $x$- and $y$-values and ask Are these positive or negative values, as represented here? Students should recognize that both are positive, so teachers should use that to point out that, in Point-Slope Form, the positive $y$ and positive $x$ should be left alone – that is, the $x_2$ and $y_2$ should be left alone. Thus, the $x$- and $y$-values of the point should be plugged in for $x_1$ and $y_1$. For this situation, that would mean $y_2 - y_1 = m(x_2 - x_1) \Rightarrow y_2 - 2800 = 340.75(x_2 - 7.8)$ or $y - 2800 = 340.75(x - 7.8)$.

But what does this mean? A student is apt to point out that we still don’t see the $y$-intercept $(142.15)$. The teacher should ask students to solve the equation for $y$:

\[
\begin{align*}
  y - 2800 &= 340.75x - 2657.85 \quad \text{(by distributive property)} \\
  + 2800 &= + 2800 \\
  y &= 340.75x + 142.15 \Rightarrow \text{This should look familiar}
\end{align*}
\]

At this point, students can see how to use Point-Slope Form to get to Slope-Intercept Form. To answer the question asked initially, you started with $142.15 \text{ GRD}$. Thus, students can see that they can build a Slope-Intercept Form equation from a point and a slope with Point-Slope Form. Students should be told that they can use either method (slope-intercept to solve for $b$ then plug
back in to slope-intercept or point-slope into slope-intercept), but that they need to be familiar with both. This should take about 30 minutes.

Closing

To end the lesson, student questions should be addressed and students should be provided with a set of questions (10) that ask them to write and utilize different linear equations in context.

Main EU-related concepts/activities: Different types of linear equations and further discussion of the EU and euro

Day 4

Introduction

Students will pair up (potentially groups of 3, depending on class size), discuss the answer to an assigned problem from the previous day’s assignment, and put the work and answer for that problem on the white board. Once all problems are up, students will have a chance to look them over and ask questions if they got a different answer. Students should be encouraged to discuss more with each other directly rather than routing it through the teacher, though the teacher should be ready to jump in if a question goes beyond what students can reason. This introduction will take 15-20 minutes.

Instruction

Yesterday, we had closed with finding linear equations when given a slope and y-intercept and when given a point and a slope. But are there other situations to be investigated. To get this started, have students get back into the groups of 2-3 from Day 2 for the website review and Slides questions. Students will open up a new slide, make a copy and rename it with their member names, and then investigate the sites listed on slide 1 (see list below). These sites discuss EU nations that do not use the euro. After reviewing the information provided, they will answer the questions posed on the remaining slides:

Slide 2: What countries inside the EU do not use the euro?
Slide 3: Why do these countries do not use the euro?
Slide 4: How do countries transition to adopting the euro?
Slide 5: What are your thoughts or takeaways from this?

Groups should be given ~20 minutes to work through this, then come back together to discuss answers, which may take an additional 10 minutes. The teacher should ensure that students understand the different reasons countries have not adopted the euro before proceeding.

Now, pose the following to students:

A friend of mine recently finished a job in Sweden. She told me about her pay by saying that in a 30-hr work week, she made 594 euros, while during a 52-hr week, she made 985.60 euros. But living in Sweden, she was actually paid in Swedish krona (SEK). She always said that she gave the amount in euros because it was easier for her parents to equate the amounts. What was my
friend’s base pay and hourly rate of pay in SEK? Was she correct in believing that the euro is closer in value to the dollar than the krona?

Students will need some time to independently look at this and determine what they have been given. Provide students ~1-2 minutes to collect their thoughts, then have them turn and talk to a neighbor. Give these discussions ~5 minutes, then have pairs report out to the class. Anticipated answers include:

*We’ll need a rate of change/slope
*What is base pay? → to this, explain that base pay is what you start with before doing any work (like a service fee) or before any time has passed ... this will likely lead to the idea that the base pay is the y-intercept, which we also need to find
*We’ve been given a point from the hours and pay (30, 594) and a point with (52, 985.60)

The last point is critical, as students need to realize that they’re being given the equivalent of two points relating time to pay. If this is not clear, have student define variables, with x being what must come first and y being what must follow. Therefore, x = time worked and y = pay (must work before getting paid). This will help students form points.

But what to do with those points? Neither is a y-intercept (x is not 0), and a point alone does not make a slope. But remember the slope formula: \( m = \frac{y_2 - y_1}{x_2 - x_1} \). Therefore, \( m = \frac{985.60 - 594 \text{ euro}}{52 \text{ hrs} - 30 \text{ hrs}} = \frac{391.6 \text{ euros}}{22 \text{ hours}} = 17.8 \text{ euros/hr} \). This gives a rate of change, or an m-value, but not a y-intercept. However, keep in mind that we have a slope and two points to choose from. Pick any point (doesn’t matter which, both will give the same answer) and use the slope in Point-Slope Form to find the y-intercept.

\[
\begin{align*}
\ y - 594 &= 17.8(x - 30) \text{ or } y - 985.60 = 17.8(x - 52) \\
\ y - 594 &= 17.8x - 534 \text{ or } y - 985.60 = 17.8x - 925.60 \text{(by Distributive Property)} \\
\ y + 594 &= 17.8x + 594 \text{ or } y + 985.60 = 17.8x + 985.60 \\
\ y &= 17.8x + 60 \text{ or } y = 17.8x + 60
\end{align*}
\]

This provides a y-intercept of 60. In context, the y-intercept is the starting value or the base pay, so the base pay is 60 euro. The rate of change or slope is the rate of hourly pay, which is 17.8 euro/hr. But how do we convert these to Swedish krona?
Have students go to Google and search ‘euro to SEK’. This will give a reasonably current exchange rate. As students to use this exchange rate to convert the euro amounts to SEK. For example, at the time of writing, the exchange rate is 1 SEK is .097 euro. Therefore:

\[
\frac{17.8 \text{ euro}}{hr} \times \frac{1 \text{ SEK}}{.097 \text{ euro}} = 183.50 \text{ SEK} \leftarrow \text{Hourly rate in Swedish krona}
\]

\[
60 \text{ euro} \times \frac{1 \text{ SEK}}{.097 \text{ euro}} = 618.56 \text{ SEK} \leftarrow \text{Base pay in Swedish krona}
\]

Slope-Intercept Equation: \( y = 183.50x + 618.56 \)

Students will likely remark on the discrepancy between the amounts. So, in looking at the connection to the dollar, have students Google search ‘euro to USD’ and ‘SEK to USD’. Allow students to compare which is two are closer in value and discuss what that represents. This final discussion and conversation should take ~30 minutes.

**Closing**

With the time remaining in class, students should begin working on problems that incorporate different aspects of all scenarios (slope and y-intercept, a point and a slope, two points) in different contextual problems (~10). These problems should begin to incorporate scenarios outside of monetary conversions to expand the scope and applicability of this skill set.

Main EU-related concepts/activities: Further discussion of linear equations and EU nations that do not use the euro (and why)

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**Day 5**

**Introduction**

The first 35-40 minutes of class will be spent reviewing the different aspects of linear equations and how to interpret the different parts of the problems. This will include reviewing the previous day’s assignment and working through some additional contextual examples.

**Instruction**

Students will quiz on linear functions in context. Information regarding the EU will appear in the contexts, and students will be asked to utilize what they have discussed and learned in assessing different aspects of some questions. The students will be allowed the time remaining in class after the review to complete the quiz.

Main EU-related concepts/activities: Summary of the unit activities and material

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**Resources and Materials – Day 1**

- Pipe cleaners (43, same color)
- Poker chips or other disk-like manipulatives (135, same color)
- Popsicle sticks (65)
- Toothpicks (500, same color)
- Dice (90, same color)
- Measuring tape
- Scissors
- Chromebooks with internet access
- Self-stick wall easel pads
- Graph paper
- Notebook paper
- Markers
- Construction paper hats in white, red, black, yellow, green, and blue

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